11: The Unit Circle and Angle Measure

Key Terms

- **Angle:** two rays extending from a single point called the vertex.
- **Central angle:** an angle whose vertex is at the center of a circle.
- **Complementary angles:** two acute angles whose measures add up to 90° or π/2 radians.
- **Degree:** a unit of angle measure equal to 1/360 revolution of a circle.
- **Quadrantal angle:** an angle where the terminal side lies on the *x*-axis or *y*-axis.
- **Radian:** a unit of angle measure where 2π radians equals 360°.
- **Standard position:** when an angle is drawn with its initial side on the positive *x*-axis and its terminal side on a radius *r* of a circle.
- **Supplementary angles:** two angles whose measures add up to 180° or π radians.
- **Unit circle:** a circle centered at the origin of the coordinate plane with radius 1.

Key Formulas

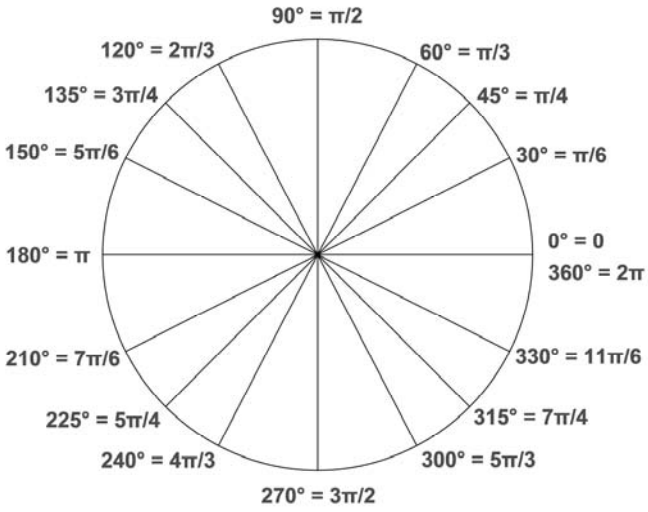
- Arc length *s*, where *r* is the radius and θ is the central angle intersecting the arc: (θ is measured in radians)

$$s = r\theta$$

- Area of a sector, where *r* is the radius and θ is the central angle of the sector: (θ is measured in radians)

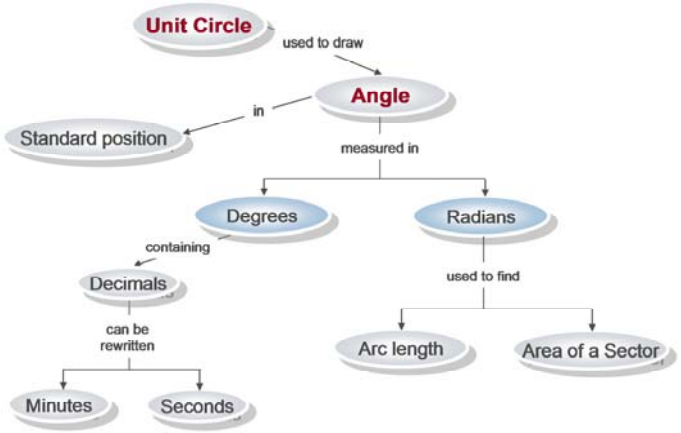
$$A = \frac{1}{2}r^2\theta$$

The Unit Circle



- Equation of the unit circle: $x^2 + y^2 = 1$
- Circumference of the unit circle: 2π units
- Area of the unit circle: π square units
- Radius of the unit circle: 1 unit
- Positive angles are measured counterclockwise on the unit circle.
- Negative angles are measured clockwise on the unit circle.

Concept map



Conversion Formulas

- **Degrees to Radians:** $\text{degrees} \cdot \frac{\pi}{180^\circ} = \text{radians}$
- **Radians to Degrees:** $\text{radians} \cdot \frac{180^\circ}{\pi} = \text{degrees}$
- **Degrees to Minutes:** $\text{minutes} = 60 \cdot \text{degrees}$
- **Minutes to Degrees:** $\text{degrees} = \frac{\text{minutes}}{60}$
- **Minutes to Seconds:** $\text{seconds} = 60 \cdot \text{minutes}$
- **Seconds to Minutes:** $\text{minutes} = \frac{\text{seconds}}{60}$

Example: Convert Radians to Degrees

Find the degree measure that is equivalent to an angle that measures $\frac{3\pi}{4}$ radians.

Solution:

$$\text{radians} \cdot \frac{180^\circ}{\pi} = \text{degrees}$$

$$\frac{3\pi}{4} \cdot \frac{180^\circ}{\pi} = \text{degrees}$$

$$\frac{3 \cdot 180^\circ}{4} = \text{degrees}$$

$$135^\circ = \text{degrees}$$

Example: Arc Length

Find the length of the arc created by a circle of radius 2 and a central angle measuring $\frac{\pi}{6}$.

Solution: Use the formula for arc length, $s = r\theta$.

$$r = 2 \qquad \theta = \frac{\pi}{6}$$

Insert the given values into the formula.

$$s = 2 \cdot \frac{\pi}{6} = \frac{2\pi}{6} = \frac{\pi}{3}$$

How to Use This Cheat Sheet: These are the key concepts related this topic. Try to read through it carefully twice then write it out on a blank sheet of paper. Review it again before the exams.